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Mathematics: analysis and approaches

Higher level

Paper 1

8 May 2023

Zone A afternoon | Zone B morning | Zone C afternoon

Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.



Please **do not** write on this page.

Answers written on this page
will not be marked.



7. [Maximum mark: 6]

Consider $P(z) = 4m - mz + \frac{36}{m}z^2 - z^3$, where $z \in \mathbb{C}$ and $m \in \mathbb{R}^+$.

Given that $z - 3i$ is a factor of $P(z)$, find the roots of $P(z) = 0$.

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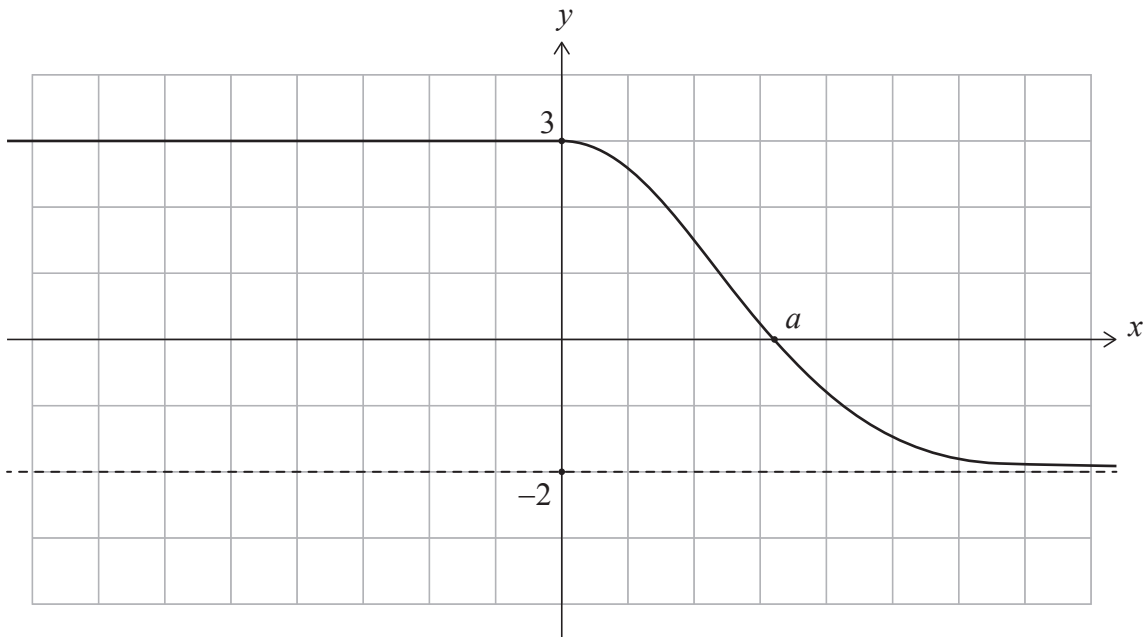
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Turn over

8. [Maximum mark: 7]

Part of the graph of a function, f , is shown in the following diagram. The graph of $y = f(x)$ has a y -intercept at $(0, 3)$, an x -intercept at $(a, 0)$ and a horizontal asymptote $y = -2$.



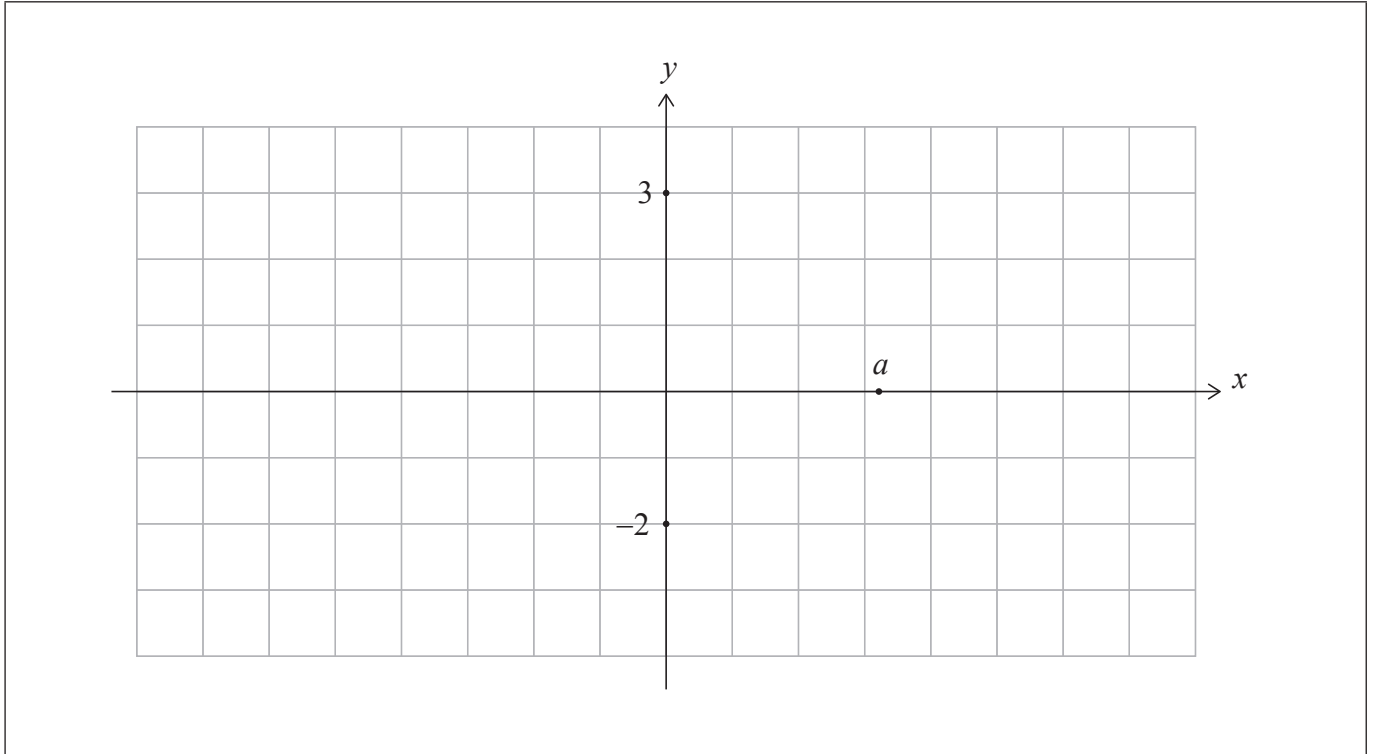
(This question continues on the following page)



(Question 8 continued)

Consider the function $g(x) = |f(|x|)|$.

- (a) On the following grid, sketch the graph of $y = g(x)$, labelling any axis intercepts and giving the equation of the asymptote. [4]



- (b) Find the possible values of k such that $(g(x))^2 = k$ has exactly two solutions. [3]

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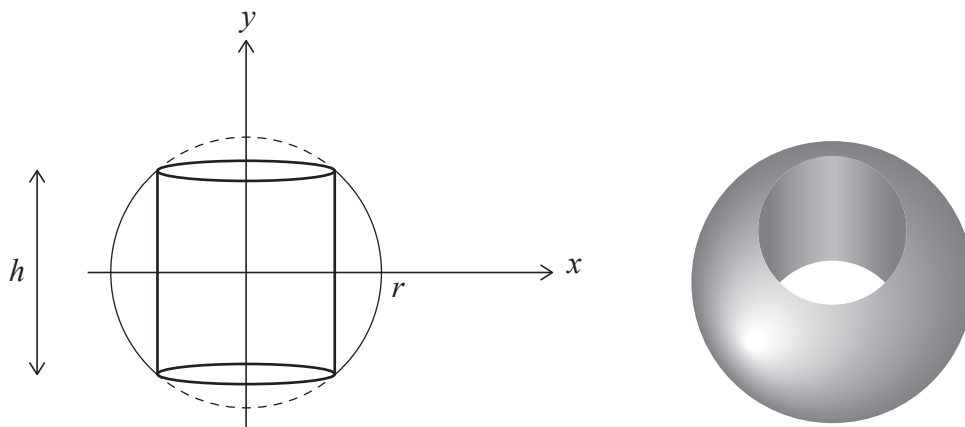
9. [Maximum mark: 7]

The function f is defined by $f(y) = \sqrt{r^2 - y^2}$ for $-r \leq y \leq r$.

The region enclosed by the graph of $x = f(y)$ and the y -axis is rotated by 360° about the y -axis to form a solid sphere. The sphere is drilled through along the y -axis, creating a cylindrical hole. The resulting spherical ring has height, h .

This information is shown in the following diagrams.

diagram not to scale



The spherical ring has a volume of π cubic units. Find the value of h .

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 14]

Consider the arithmetic sequence u_1, u_2, u_3, \dots .

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

- (a) (i) Find the sum of the first five terms. [4]
- (ii) Given that $S_6 = 60$, find u_6 . [4]
- (b) Find u_1 . [2]
- (c) Hence or otherwise, write an expression for u_n in terms of n . [3]

Consider a geometric sequence, v_n , where $v_2 = u_1$ and $v_4 = u_6$.

- (d) Find the possible values of the common ratio, r . [3]
- (e) Given that $v_{99} < 0$, find v_5 . [2]

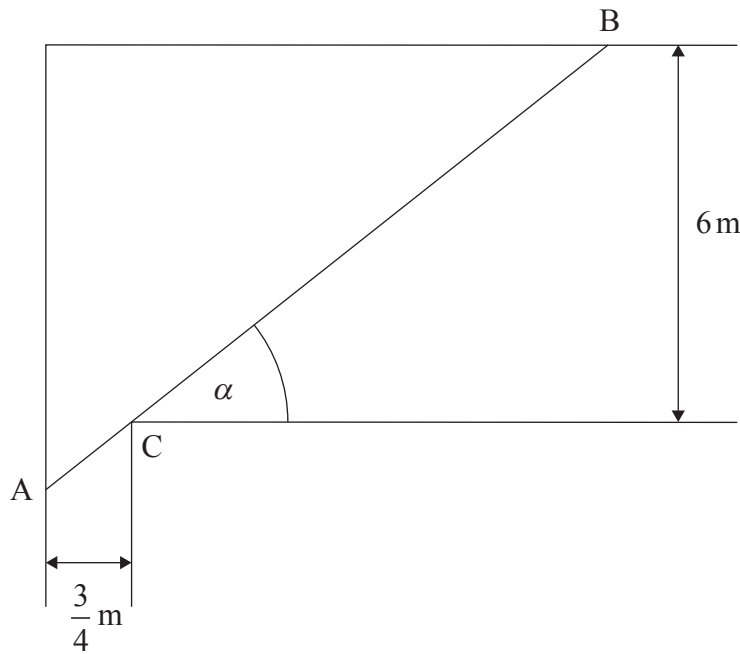


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11. [Maximum mark: 19]

Consider the following diagram, which shows the plan of part of a house.

diagram not to scale



A narrow passageway with width $\frac{3}{4}$ m is perpendicular to a room of width 6 m. There is a corner at point C. Points A and B are variable points on the base of the walls such that A, C and B lie on a straight line.

Let L denote the length AB in metres.

Let α be the angle that [AB] makes with the room wall, where $0 < \alpha < \frac{\pi}{2}$.

(a) Show that $L = \frac{3}{4} \sec \alpha + 6 \operatorname{cosec} \alpha$. [2]

(b) (i) Find $\frac{dL}{d\alpha}$.

(ii) When $\frac{dL}{d\alpha} = 0$, show that $\alpha = \arctan 2$. [5]

(This question continues on the following page)



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(Question 11 continued)

- (c) (i) Find $\frac{d^2L}{d\alpha^2}$.
- (ii) When $\alpha = \arctan 2$, show that $\frac{d^2L}{d\alpha^2} = \frac{45}{4}\sqrt{5}$. [7]
- (d) (i) Hence, justify that L is a minimum when $\alpha = \arctan 2$.
- (ii) Determine this minimum value of L . [3]

Two people need to carry a pole of length 11.25 m from the passageway into the room. It must be carried horizontally.

- (e) Determine whether this is possible, giving a reason for your answer. [2]

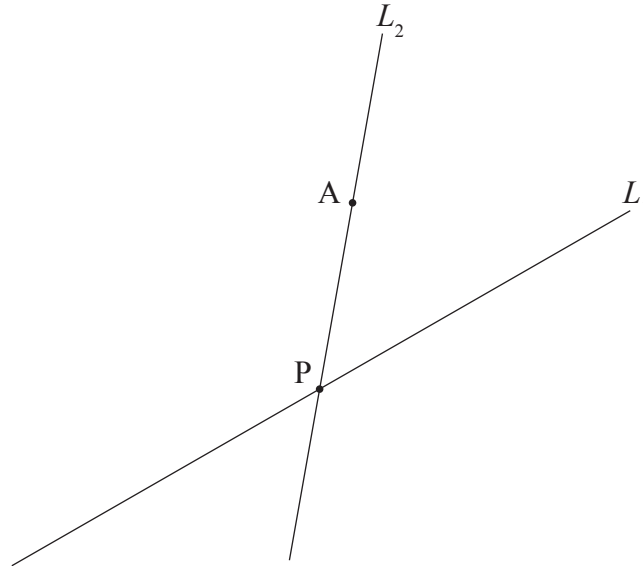


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12. [Maximum mark: 21]

Two lines, L_1 and L_2 , intersect at point P. Point A($2t$, 8, 3), where $t > 0$, lies on L_2 . This is shown in the following diagram.

diagram not to scale



The acute angle between the two lines is $\frac{\pi}{3}$.

The direction vector of L_1 is $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\vec{PA} = \begin{pmatrix} 2t \\ 0 \\ 3+t \end{pmatrix}$.

- (a) Show that $4t = \sqrt{10t^2 + 12t + 18}$. [4]
- (b) Find the value of t . [4]
- (c) Hence or otherwise, find the shortest distance from A to L_1 . [4]

A plane, Π , contains L_1 and L_2 .

- (d) Find a normal vector to Π . [2]

The base of a right cone lies in Π , centred at A such that L_1 is a tangent to its base. The volume of the cone is $90\pi\sqrt{3}$ cubic units.

- (e) Find the two possible positions of the vertex of the cone. [7]

References:

